A “RUNNING” GRAVITATIONAL CONSTANT ?

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Abstract:

If the gravitational interaction is unified with the electro-weak and strong interactions at a mass $M = 10^{15}$ GeV, the evolution of Newton's constant must differ from its classical (general relativistic) form. We can model such behavior by introducing an ad hoc dependence on $\ell n(s/4m^2)$, where s is the usual cm energy between two protons. We can then predict the observable effects for relativistic collisions ($\sqrt{s} \sim 1.4 \times 10^4$ GeV) as well as for the case of low velocity motion ($\beta^2 \sim 10^{-5}$).

It is well known that the dimensionless coupling constants of the three gauge groups of the standard model $SU(3) \times U(2) \times U(1)$ vary with the momentum transfer of the interaction [1]. This effect which is due to the polarization of the vacuum was first recognized for the electromagnetic field. It is most prominent in the case of the color field and leads to assymptotic freedom.

Extrapolation to higher energies is governed by the equations of the renormalization group, and it is customary to consider the inverse coupling constants

$$\frac{1}{\alpha_1(\sqrt{s})} = \frac{1}{\alpha_e(m_Z)}\frac{3}{5}\cos^2\theta_W - \frac{1}{12\pi}(4n_f)\ell n\left(\frac{s}{m_Z^2}\right)$$

$$\frac{1}{\alpha_2(\sqrt{s})} = \frac{1}{\alpha_e(m_Z)}\sin^2\theta_W + \frac{1}{12\pi}\left(22 - 4n_f - \frac{1}{2}\right)\ell n\left(\frac{s}{m_Z^2}\right)$$

$$\frac{1}{\alpha_3(\sqrt{s})} = \frac{1}{\alpha_s(m_Z)} + \frac{1}{12\pi}\left(33 - 3n_f\right)\ell n\left(\frac{s}{m_Z^2}\right)$$

The above expressions have been normalized at a cm energy $\sqrt{s} = m_Z$ where the couplings are given by

$$\alpha_e = \frac{e^2}{(4\pi\epsilon_0)\hbar c} = \frac{1}{128}$$

$$\alpha_3 = \alpha_s = \frac{g_s^2}{\hbar c} = 0.118$$

$$\sin^2\theta_W(m_Z) = 0.2315$$

and n_f is the number of quark/lepton families.

The inverse couplings are plotted in Fig.1a as a function of $\sqrt{s}$. As observed by Georgi and Glashow [2] all three couplings seem to reach the same value at an energy $\sqrt{s} \simeq 10^{14}$ GeV which is referred to as the "Grand Unification Scale". If the couplings evolve according to the minimal supersymmetric model (MSSM) much better agreement is obtained, and within present uncertainties, the constants meet exactly at $\sqrt{s} = 10^{15}$ GeV, as shown in Fig.1b. [3].

The gravitational constant depends on the interaction energy as well. Consider two protons moving against each other in the laboratory frame with velocity β and energy γm_p. The gravitational coupling in this case takes the form

$$\alpha_G(\sqrt{s}) = \frac{G_N m_p^2}{\hbar c}(2\gamma^2 - 1) \tag{1}$$

The factor of $2\gamma^2$ arises because both energy γm_p and momentum $\gamma\beta m_p$ couple; see for instance [4]. The c.m. collision energy is $s = 4\gamma^2 m_p^2$, so Eq.(1) can be written as

$$\alpha_G(\sqrt{s}) = \frac{G_N m_p^2}{\hbar c}\left[\frac{s}{2m_p^2} - 1\right] \tag{2}$$

valid for $s \geq 4m_p^2$. Numerically

$$\frac{G_N m_p^2}{\hbar c} = \left(\frac{m_p}{M_P}\right)^2 = 0.59 \times 10^{-38} \tag{3}$$

where M_p is the Planck Mass. When $\sqrt{s/2} = M_p$, then α_G becomes unity.

If all four forces can be derived from a single gauge group then the gravitational coupling should reach the common value

$$1/\alpha_G = 1/\alpha_1 = 1/\alpha_2 = 1/\alpha_3 \simeq 42 \tag{4}$$

at the unification scale $\sqrt{s} = 10^{15}$ GeV. [5]. We can achieve this by modifying the classical evolution of $\alpha_G(s)$ by a logarithmic term, of the form

$$\alpha_G(\sqrt{s}) = \left(\frac{m_p}{M_P}\right)^2 \left[\left(\frac{s}{2m_p^2}\right)^{1+b\ell n(s/4m^2)} - 1\right] \tag{5}$$

The coefficient b is found, by the requirement of Eq.(4), to have the value

$$b = 0.00340 \tag{6}$$

We can now obtain the gravitational coupling at any given interaction energy. There are two possibilities for testing this hypothesis:

In the first case one tries to measure the gravitational effect at LHC energies. Here there is a significant difference in the couplings

$$\alpha_{GC}(\sqrt{s} = 1.4 \times 10^4 \text{ GeV}) \quad = \quad 6.6 \times 10^{-31} \qquad \text{classical}$$

$$\alpha_{GR}(\sqrt{s} = 1.4 \times 10^4 \text{ GeV}) \quad = \quad 2.0 \times 10^{-30} \qquad \text{running}$$

However the gravitational effects are extremely small and are dominated by the much larger electromagnetic force [6]. New detector technology [7] may make it worthwhile to re-examine such experiments.

The other possibility is to test the deviation from the classical behavior at low energies but with macroscopic bodies. The classical correction in this case leads to

$$\alpha_{GC}(\beta) = \left(\frac{m_p}{M_P}\right)^2 (1 + 2\beta^2) \tag{7}$$

while for the model of Eq.(5)

$$\begin{aligned} \alpha_{GR}(\beta) &\simeq \left(\frac{m_p}{M_P}\right)^2 \left\{\left[2(1+\beta^2)\right]^{1+b\beta^2} - 1\right\} \\ &\simeq \alpha_{GC}(\beta)[1+x] \end{aligned} \tag{8}$$

with

$$x = 2^{1+b\beta^2} - 2 \sim \sqrt{2}b\beta^2 \tag{9}$$

For a satellite in earth orbit

$$\beta^2 = 0.7 \times 10^{-9}$$

whereas for a close solar orbit

$$\beta^2 = 0.2 \times 10^{-5}$$

In this latter case, the effect of the running coupling constant is

$$x \simeq 10^{-8}$$

which could be measurable as a difference in the predicted orbital dynamics, beyond the effects of classical general relativity [8].

References

1. M.E. Peskin and D.V. Schroeder "An Introduction to Quantum Field Theory", Addison-Wesley (1995).

2. H. Georgi and S.L. Glashow, Phys. Rev. Lett. 32, 438 (1974).

3. F. Wilczek in "Critical Problems in Physics", Princeton University Press (1997).

4. A.C. Melissinos, Il Nuovo Cimento, 62B, 190 (1981).

5. I heard of this suggestion in a talk by F. Wilczek at the symposium in honor of N.P. Samios held at BNL, May 2002.

6. P. Reiner et al., Physics Letters B176, 233 (1986).

7. Ph. Bernard et al., Review of Scientific Instruments 72, 2428 (2001).

8. See for instance T.P. Krisher et al., Phys. Rev. Lett. 64, 1322 (1990).